Downlink Pilots for Hybrid Massive MIMO under Reciprocity Imperfections
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Abstract—In this paper, we study the performance of a time division duplex (TDD) hybrid massive multiple input multiple output (MIMO) system with imperfect calibration of the transmit and receive radio frequency chains. Using deterministic equivalent (DE) analysis, we establish the loss in the achievable rates due to calibration imperfections. We then discuss limited downlink training to estimate the effective downlink channel, and compare the relative merits of this method against frequency division duplexed (FDD) operation. Finally, via detailed numerical simulations, we evaluate the relative performance of no downlink training, limited downlink training, and FDD mode operation in the presence of reciprocity calibration errors.

Index Terms—Massive MIMO, channel reciprocity calibration, deterministic equivalents

I. INTRODUCTION

Cellular communication systems simultaneously serving a large number of user equipments (UEs) over the same time-frequency resource by employing a large number of base station (BS) antennas, popularly known as massive MIMO systems are a cornerstone of next generation wireless communication systems [1]. Massive MIMO systems, since their introduction about a decade ago [1], [2], have received much research interest due to their potential advantages in terms of high spectral and energy efficiencies [1], linear signal processing, and near AWGN behaviour of the effective propagation channels. Traditional massive MIMO architectures assume dedicated radio-frequency (RF) chains for each of the antenna at the BS [1]. Therefore, a large number of antennas at the BS leads to a proportional increase in the cost of the RF chains, and the complexity of the digital signal processing (DSP) architecture. It has recently been proposed to keep a check on these costs by letting multiple BS antennas share the same RF chain [3]–[5]. This approach, known as hybrid beamforming, has been shown to trade off the hardware costs against the massive MIMO array gain [5]–[7]. A large number of BS antennas have been shown to mitigate the effects of finite precision quantization of a hybrid beamformer.

It has also been argued that the operation of massive MIMO systems in the conventional frequency division duplexed (FDD) mode involves prohibitively high downlink training costs, and therefore canonical massive MIMO operates in the time division duplexed (TDD) mode [1], [8]. It is shown that due to the effect of channel hardening [9], and due to the reciprocity in the propagation channels [10], TDD massive MIMO systems can operate in the absence of any downlink training, such as the demodulation reference signals (DMRS), being employed by the current generation of cellular communication systems.

However, while it is fair to assume a reciprocal air propagation channel between the transmit and the receive antennas of the BS and the UE, the channel seen by the encoder/ data detector includes the RF chains of the transmitter and the receiver, that need precise calibration to be reciprocal. The present literature on reciprocity calibrations in massive MIMO discusses both the calibration techniques for transmit and receive RF chains [11], as well as the effects of imperfections in reciprocity calibrations [12]–[14]. However, the discussion on the mitigation of the effects of reciprocity calibration imperfections is limited to [14], that discusses the effect of reciprocity traditional massive MIMO systems and proposes a blind channel estimation algorithm to counter the same. It is noted in [14], that the proposed blind channel estimation algorithm works only for uncoded transmission, requires large data frames, and is susceptible to what is known as channel corruption [15]. Therefore, it is necessary to explore other potential techniques for mitigating the effects of reciprocity imperfections in massive MIMO systems.

In this paper, we investigate the effects of reciprocity calibration imperfections on the rates achievable by hybrid massive MIMO systems, and discuss the relative merits of different downlink training approaches for mitigating these effects. Our key contributions can be listed as follows:

1) We derive the achievable rates for a hybrid massive MIMO system in the presence of reciprocity calibration errors. (See Section III.)
2) We describe a limited downlink training scheme for mitigating the effects of calibration imperfections, and derive the consequent achievable rates. We also compare these achievable rates against those of an FDD hybrid massive MIMO system. (See Section IV.)
3) We use these results to argue that it is necessary to use some form of downlink training, like the DMRS, even in the presence of channel hardening.
4) Via extensive numerical simulations, we comment on the need for the use of limited/ full downlink training for the system under study. (See Section V.)

The key message of this paper is that downlink training is necessary to nullify the effects of reciprocity imperfections in hybrid massive MIMO systems, however, the extent of downlink training required depends on the severity of calibration errors.
imperfections. We next discuss the system model considered in this paper.

II. SYSTEM MODEL

We consider a single cell massive MIMO system, with a BS equipped with \(M\) transmit RF-chains and \(N\) antennas, used to serve \(K\) (\(K \ll M \ll N\)) single antenna UEs. In this paper we first consider TDD operation with training in the uplink [1], and then compare its performance against downlink training and full TDD mode operation. The channel is assumed to be block fading, such that the total length of a data frame is smaller than the coherence time of the channel. The data frame is further divided into three subframes, viz., data transmission, uplink training, and downlink data transmission. Since the BS assumes the channel matrix to be reciprocal, the channel estimate obtained during uplink training is used to obtain the downlink precoding matrices at the BS.

The uplink channel between the \(k\)th BS antenna and the \(b\)th user's receive RF chain, and \(h_{u,ik}\) represents the fast fading component modeled as a zero mean circularly symmetric complex Gaussian (ZMCSCG) random variable with unit variance, denoted by \(CN(0,1)\). We further denote the uplink channel as \(h_{u,ik} = a_{t,k}b_{r,i}h_{ik}\), where \(a_{t,k}\) is the complex valued gain associated with the transmit RF chain of the \(k\)th user, \(b_{r,i}\) is the complex valued gain associated with the receive RF chain of the \(b\)th BS antenna, and \(h_{ik}\) is the fast fading component between the \(k\)th BS UE and the \(b\)th BS antenna.

Defining \(h_{u,ik}\) as the \((i,k)\)th entry of the matrix \(H_u\), and \(\beta = [\beta_1, \ldots, \beta_K]^T\), the uplink channel matrix between the UEs and the BS can be expressed as \(H_u\text{diag}(\sqrt{\beta}) \in \mathbb{C}^{N \times K}\). Similarly, the fast fading component of the downlink channel between the \(i\)th BS antenna and the \(k\)th UE can be represented as, \(d_{d,i,k} = a_{t,k}b_{r,i}h_{ik}\), where \(a_{t,k}\) is the complex valued gain of the \(k\)th user's receive RF chain, and \(b_{r,i}\) is the complex valued gain of the \(i\)th BS antenna’s transmit RF chain.

As we assume that both the BS and the UEs calibrate their transmit and receive RF chain using over-the-air calibration techniques in [16]. Following this calibration, the BS assumes the uplink and downlink channels to be reciprocal, and consequently assumes that the estimated downlink channel between the \(i\)th BS antenna and the \(k\)th UE, \(H_d\), is equal to the \(H_u\) to the BS and \(K\)th UE, \(H_{d,i,k}\) is equal to the available uplink channel estimate \(h_{u,ik}\). However, since the channel calibration is imperfect, the resulting estimates of the downlink channel coefficients are also erroneous.

To model the error in the channel coefficient estimates due to imperfect calibration, we define \(d_{u,i,k} = \delta_{u,i,k}e^{j\phi_{u,i,k}} \triangleq \frac{a_{t,k}b_{r,i}}{a_{t,k}}\), \(d_{b,i} = \delta_{b,i}e^{j\phi_{b,i}} \triangleq \frac{b_{r,i}}{b_{r,i}}\). Here, \(\delta_{u,i,k}\) and \(\delta_{b,i}\) are the magnitude calibration errors, and \(\phi_{u,i,k}\) and \(\phi_{b,i}\) are the phase calibration errors at the \(k\)th UE and \(i\)th BS antenna, respectively. We model both the magnitude and phase calibration errors to be i.i.d. rvs with finite moments for both the UEs and the BS. Further, we assume that the probability density function (pdf) of the phase calibration error is symmetric about zero. We also assume that the distribution of the calibration errors is agnostic to the calibration algorithm being used [12], [14], [17].

During the uplink training phase, the BS uses the pilots transmitted by the UEs to obtain the MMSE estimate of the uplink channel matrix. It is important to note that in an hybrid massive MIMO setting, each RF chain is connected to multiple BS antennas, and therefore the system can no longer be trained using \(K\) uplink pilots. Instead, we use the channel estimation technique considered in [4], wherein the users train \(M\) out of the \(N\) BS antennas during each training cycle consisting of \(K\) orthogonal pilot transmissions. It is easy to argue that this scheme, known as the round robin training scheme, entails a total of \(\lceil \frac{N}{M} \rceil\) training cycles.

Letting \(h_{u,ik}\) be the MMSE estimate of \(h_{u,ik}\), it can be shown that \(h_{u,ik} = a_k\tilde{h}_{u,ik} + \tilde{a}_k\tilde{h}_{u,ik}\), where \(h_{u,ik} \sim \mathcal{CN}(0,1)\) is the channel estimation error such that \(E[|h_{u,ik}|^2] = 0\), \(a_k = \sqrt{\frac{\beta_k}{\sigma^2 + \beta_k}}\), with \(\beta_{p,k}\) being the pilot power transmitted by the \(k\)th user, \(\beta_k\) being the macroscopic fading coefficient for the \(k\)th user, \(N_0\) being the variance of the AWGN, and \(\tilde{a} \triangleq \sqrt{1 - |a|^2}\) for any variable \(a\). Defining \(a \triangleq [a_1, \ldots, a_K]^T\) and \(\mathbf{A} = \text{diag}(a)\), the overall uplink channel matrix can be written as

\[
H_u = \tilde{H}_u \mathbf{A} + \tilde{H}_u \tilde{\mathbf{A}}. \tag{1}
\]

Incorporating the effects of reciprocity imperfections, we can write

\[
\hat{H}_d = D_d \tilde{H}_d D_u \mathbf{A} + \tilde{H}_d \tilde{\mathbf{A}}, \tag{2}
\]

with \(\tilde{h}_{u,ik} \sim \mathcal{CN}(0,1)\) being the i.i.d entries of \(\tilde{H}_d\), \(D_d\) being an \(N \times N\) diagonal matrix with \(d_{b,i}\) as its \(i\)th entry, and \(D_u\) being a \(K \times K\) diagonal matrix with \(d_{u,i}\) as its \(i\)th entry. Therefore, the estimate of the downlink channel is off from the true channel due to both the additive noise during the training phase as well as the multiplicative error due to calibration imperfections.

We know that a hybrid precoder is split into a digital precoder and an analog beamformer. The analog beamformer matrix, denoted as \(\hat{F} \in \mathbb{C}^{N \times M}\), is designed by extracting the phase of the estimated uplink channel \(\tilde{H}_u\). The first \(K\) columns of \(\hat{F}\) are chosen such that its \((p,q)\)th entry is given as

\[
\hat{F}_{p,q} = \frac{1}{\sqrt{N}} \exp(j\hat{\theta}_{p,q}), \tag{3}
\]

where, \(\hat{\theta}_{p,q}\) is the phase of the \((p,q)\)th entry of \(\hat{H}_u\). The phase error due to finite precision quantization at the analog beamformer can be safely absorbed into the calibration imperfections in the RF chains. The entries of the remaining \((M-K)\) columns are \(K\), phases of the entries \((M-K)\) columns of \(\hat{F}\) randomly chosen. The equivalent low dimensional downlink channel, as seen by the digital precoder can be expressed as \(\tilde{H}_u^H \hat{F} \in \mathbb{C}^{K \times M}\).

Now, the BS uses the available estimate of the low dimensional equivalent channel to design the precoding matrix. For maximal ratio transmission (MRT), the precoding matrix can be given as,

\[
\hat{W} \triangleq \hat{F}^H \tilde{H}^*_u = \tilde{F}^H \tilde{H}_u^*. \tag{4}
\]

III. ACHIEVABLE RATE ANALYSIS WITH NO DOWNLINK TRAINING

Letting \(x_{d,l}[n]\) denote the downlink symbol meant for the \(l\)th user, we can write the signal received by the \(k\)th user as
Under perfect reciprocity calibration, the effective downlink channel coefficient. Consequently, the array gain advantage of massive MIMO.

\[ y_{d,k}[n] = \alpha_{k} d_{a,k} \sqrt{\beta_{k}} P_{d,k} \frac{\bar{h}_{d,k}^T D_b \bar{F}^H \bar{h}_{d,k}^* x_{d,k}[n]}{M} + \bar{a}_{k} \sqrt{\beta_{k}} P_{d,k} \frac{\bar{h}_{d,k}^T D_b \bar{F}^H \bar{h}_{d,k}^* x_{d,k}[n]}{M} \]

\[ + \sum_{l=1, l \neq k}^{K} \sqrt{\beta_{k}} P_{d,l} \frac{\bar{h}_{d,l}^T D_b \bar{F}^H \bar{h}_{d,l}^* x_{d,l}[n]}{M} + \sqrt{N_0 w_k}, \quad (5) \]

where \( P_{d,k} \) is the transmit power allocated to the \( k \)th user, such that \( \sum_{k=1}^{K} P_{d,k} = P_{d,s} \), with \( P_{d,s} \) being the total allowed downlink power. It has been shown in [10] that for massive MIMO downlink systems employing MRT, the optimal power control strategy is to uniformly distribute the transmit power across all the BS RF chains. In the above, the first term corresponding to the desired signal at the \( k \)th user can be expanded as,

\[ a_{k} d_{a,k} \sqrt{\beta_{k}} P_{d,k} \frac{\bar{h}_{d,k}^T D_b \bar{F}^H \bar{h}_{d,k}^*}{M} = a_{k} d_{a,k} \sqrt{\beta_{k}} P_{d,k} \frac{\bar{h}_{d,k}^T D_b \bar{F}^H \bar{h}_{d,k}^*}{M} \]

\[ = a_{k} d_{a,k} \sqrt{\beta_{k}} P_{d,k} \frac{\bar{h}_{d,k}^T D_b \left( \sum_{m=1}^{M} \tilde{f}_m \hat{H}_m \right) \bar{h}_{d,k}^*}{M}, \quad (6) \]

with \( \tilde{f}_m \in \mathbb{C}^N \) being the \( m \)th column of the analog beamforming matrix \( \tilde{F} \), such that its \( i \)th element is denoted as \( f_{m,i} \). We can therefore write,

\[ \hat{h}_{d,k}^T D_b \bar{F}^H \bar{h}_{d,k}^* = M \frac{N}{M} \sum_{i=1}^{N} d_{b,i} |\tilde{h}_{d,k,i}|^2, \quad (7) \]

with \( d_{b,i} \) being the \( i \)th diagonal entry of \( D_b \). Now, invoking results from random matrix theory [9], it is easy to show that

\[ \frac{1}{M} \sum_{i=1}^{N} d_{b,i} |\tilde{h}_{d,k,i}|^2 \rightarrow E[|\delta_b \cos \phi_b|^2] a.s. 0. \]

Also,

\[ \frac{1}{M} \left| \tilde{h}_{d,k}^T D_b \bar{F}^H \bar{h}_{d,k}^* \right|^2 \rightarrow E[|\delta_b \cos \phi_b|^2] a.s. 0. \]

Under perfect reciprocity calibration,

\[ \frac{1}{M} \left| \tilde{h}_{d,k}^T D_b \bar{F}^H \bar{h}_{d,k}^* \right|^2 - 1 \rightarrow a.s. 0. \]

\[ \frac{1}{M} \left| \tilde{h}_{d,k}^T D_b \bar{F}^H \bar{h}_{d,k}^* \right|^2 - 1 \rightarrow a.s. 0. \]

and

\[ \frac{1}{M} \left| \tilde{h}_{d,k}^T D_b \bar{F}^H \bar{h}_{d,k}^* \right|^2 - 1 \rightarrow a.s. 0. \]

Now, in the absence of downlink training, the users assume perfect reciprocity calibration, and use the expected value of the downlink channel gain with no calibration errors as the effective downlink channel coefficient. Consequently, the channel coefficient of the desired signal, used for decoding is biased, and SINR achievable by the \( k \)th user can be expressed as (8).

Note that since both the numerator and the denominator of (8) scale with the number of BS RF chains, the achievable SINR, and consequently, the achievable rate saturates, nullifying the array gain advantage of massive MIMO.

The average achievable rate in this case is for an overall frame duration \( T \) given as,

\[ R_k^N = \frac{T - K \sqrt{\frac{\mathcal{N}}{M}} \log_2(1 + \gamma_k^N)}{T}. \quad (9) \]

IV. LIMITED DOWNLINK TRAINING

Since the SINR saturation occurs due to a mismatch in the gain and phase of the effective downlink channel and its expected value, the BS can either transmit downlink pilots over the effective channels to let the UEs obtain their estimates, or the system can operate in the FDD mode.

In case the BS transmits limited downlink pilots over the \( K \) effective downlink channels, over \( K \) time slots, the pilot signal received by the \( k \)th user can be expressed as

\[ y_{d,p,k} = \sqrt{\gamma_k} P_{d,p,k} \left( a_{d,u,k} \tilde{h}_{d,k}^T D_b + \bar{a}_{d,k} \hat{h}_{d,k}^T \right) \bar{F}^H \bar{h}_{d,k}^* \]

\[ + \sqrt{N_0 w_k} = g_{kk} \sqrt{P_{d,p,k}} + \sqrt{N_0 w_k}, \quad (10) \]

with \( g_{kk} \) being the effective downlink channel for the \( k \)th user’s data stream, and \( P_{d,p,k} \) the effective downlink pilot power. Now, under the assumption of perfect reciprocity, and for a large number of BS antennas, \( g_{kk} \approx E[|g_{kk}|] = M a_k \sqrt{\beta_k} \), that is the value assumed at the UE while applying the use and forget technique. In this case, the pilot signal received at the \( k \)th user should ideally take the value,

\[ \hat{y}_{d,p,k} = M a_k \sqrt{\beta_k} P_{d,p,k}. \quad (11) \]

Since we are interested in estimating the difference between the true channel and its assumed value at the \( k \)th UE, represented as \( \hat{g}_{kk} = g_{kk} - M a_k \sqrt{\beta_k} \). Also, defining \( \hat{y}_{d,p,k} \triangleq \hat{y}_{d,p,k} - \hat{y}_{d,k} \), it is easy to show that

\[ \hat{y}_{d,p,k} \rightarrow g_{kk} \sqrt{P_{d,p,k}} + \sqrt{N_0 w_k}, \]

and therefore, \( \hat{y}_{d,p,k} \) can be used to obtain the MMSE estimate, \( \hat{g}_{kk} \), of \( g_{kk} \), which can further be used to write the estimate \( \hat{g}_{kk} \) of \( g_{kk} \) as \( \hat{g}_{kk} \approx E[|g_{kk}|] + \hat{g}_{kk} \), such that

\[ \hat{g}_{kk} = c_k \hat{y}_{d,k} + \hat{g}_{kk}, \]

where \( \hat{g}_{kk} \) is the channel estimation error, uncorrelated with \( \hat{g}_{kk} \), and

\[ \hat{g}_{kk} = \frac{N_0}{M a_k^2 \beta_k P_{d,p,k} (1 - |\delta_b \cos \phi_b|^2)^2 + N_0}. \]

Using analysis similar to the case with no downlink pilot transmission, it can be shown that the SINR achievable at the \( k \)th user in this case can be written as (12). We note that the SINR still saturates as a function of the number of BS RF chains. However, the denominator term now depends on \( \hat{c}_k \), that can be made arbitrarily small using higher downlink pilot power.

The achievable rate in this case can be given as,

\[ R_k^D = \frac{T - K \sqrt{\frac{\mathcal{N}}{M}} \log_2(1 + \gamma_k^D)}{T}. \quad (13) \]

V. FDD MODE OPERATION

Alternatively, the BS can use full downlink training, involving transmission of downlink pilots, each having an energy
\[ \gamma^N_k = \frac{MP_{d,s,k}\beta_k a_k^2}{P_{d,s,k}\beta_k(a_k^2 + Ma_k^2(1 - d_{u,k}^2)E^2[\delta_k \cos \phi_k]) + \sum_{i=1, i \neq k}^{K} P_{d,s,i}\beta_k + N_0} \xrightarrow{a.s.} 0, \] (8)

\[ \gamma^D_k = \frac{MP_{d,s,k}\beta_k a_k^2}{P_{d,s,k}\beta_k(a_k^2 + Ma_k^2(1 - d_{u,k}^2)E^2[\delta_k \cos \phi_k]) + \sum_{i=1, i \neq k}^{K} P_{d,s,i}\beta_k + N_0} \xrightarrow{a.s.} 0, \] (12)

\[ \mathcal{E}_{d,p} \text{ from all the } N \text{ BS antennas over } N \text{ training slots. These pilots are used by the UEs to obtain estimates of the downlink channel coefficients, that are communicated to the BS over a } Q \text{ bit rate limited channel such that } h_{d,ik}, \text{ can be expressed in terms of the channel estimate } \tilde{h}_{u,ik} \text{ as} \]

\[ h_{d,ik} = e_k \tilde{h}_{d,ik} + \tilde{e}_k \tilde{h}_{d,ik} \] (14)

\[ \tilde{h}_{d,ik} \text{ representing the ZMCSCG distributed error due to estimation inaccuracies and quantization noise, such that } E[\tilde{h}_{d,ik}\tilde{h}_{d,ik}^*] = 0. \text{ Also, it is easy to show that } [10], \]

\[ \tilde{e}_k = \sqrt{(1 - 2^{-Q})\frac{N_0}{\beta_k e_{p,k} + N_0}} + 2^{-\frac{Q}{2}}. \] (15)

Again the BS uses the available estimates to obtain the analog and digital beamforming matrices, and transmit the data symbols to the UEs. Again letting \( x_{d,l}[n] \) denote the downlink data symbol that the \( l \)th user, the signal received by the \( k \)th user at the \( n \)th instant can be expressed as,

\[ y_{d,k}[n] = e_k \sqrt{\frac{P_{d,s,k}}{M}} \tilde{h}_{d,k}^T \hat{F} \hat{F}^H \tilde{h}_{d,k} x_{d,k}[n] \]

\[ + \tilde{e}_k \sqrt{\frac{P_{d,s,k}}{M}} \tilde{h}_{d,k}^T \hat{F} \hat{F}^H \tilde{h}_{d,k}^* x_{d,k}[n] + \sqrt{N_0 w_k}. \] (16)

Again using DE analysis, we can write

\[ e_k \sqrt{\frac{P_{d,s,k}}{M}} \tilde{h}_{d,k}^T \hat{D} \hat{D}^H \tilde{h}_{d,k} x_{d,k}[n] - M e_k \sqrt{\frac{1}{\beta_k P_{d,s,k}}} \xrightarrow{a.s.} 0, \]

that is the value available at the UEs. Consequently, the achievable SINR for the 4th UE takes the form,

\[ \gamma^F_k = \frac{MP_{d,s,k}\beta_k e_k^2}{\tilde{e}_k P_{d,s,k}\beta_k + \sum_{i=1, i \neq k}^{K} P_{d,s,i}\beta_k + N_0} \xrightarrow{a.s.} 0. \] (17)

It can be observed that in this case, while the numerator term grows linearly with the number of BS RF chains, no such behavior is shown by the noise and interference terms in the denominator, thus indicating true massive MIMO array gain. We can now write the achievable rate with the system operating in the FDD mode as,

\[ R_k^F = \frac{T - N}{T} \log_2(1 + \gamma^F_k). \] (18)

It can therefore be seen that in case of reciprocity imperfections, a true massive MIMO like array gain can only be achieved at the cost of full downlink training resulting in longer training durations and shorter data transmission durations. In the next section, we compare the relative performance of limited downlink training against full FDD mode operation via numerical simulations.

VI. SIMULATION RESULTS

In this section, we validate the results derived in the previous sections via Monte-Carlo simulations, obtain further insights into the behavior of hybrid massive MIMO systems with and without reciprocity imperfections using numerical methods. We consider a single cell narrowband massive MIMO system with \( N = 1024 \) antenna BS serving 8 users, over a frame duration of \( T = 2048 \) channel uses. We assume the system to operate at a carrier frequency of 2 GHz, and a bandwidth of 1 MHz. For all the experiments, we assume a path loss inversion based uplink pilot power control, such that the pilot SNR at the BS is 10 dB for all the users. The downlink transmit SNR at the BS is also assumed to be 10 dB.

In Fig. 1 we plot the achievable rate as a function of the number of BS RF chains for different values of phase error \( \phi_0 \), where the phase error \( \phi_0 \) is distributed uniformly over the support \([-\phi_0, \phi_0]\). We observe that an increased phase error results in a significant worsening and eventual saturation of the achievable rate for the hybrid massive MIMO system under study, as predicted by (8).

In Fig 2 we plot the achievable rates as a function of the number of BS RF chains, with a uniformly distributed phase error between -25 and 25 degrees, and different values of downlink pilot SNR. We observe that for a sufficiently large downlink pilot SNR, we obtain a curve parallel to the no phase
error plot, thus affirming the restoration of the array gain for massive MIMO systems.

Finally in Fig. 3 we compare the pilot based downlink training against full FDD mode operation, and observe that the additional training costs involved in full FDD mode operation result in a substantial reduction in the achievable rates, while TDD mode operation with limited downlink offers significantly larger achievable rates.

VII. Conclusions and Future Work

In this paper we have analyzed the performance of a hybrid massive MIMO system under imperfect RF chain reciprocity calibrations, and observed the effects of magnitude and phase calibration errors on the achievable rates. Following this, using limited downlink training, and full FDD like training, we have shown the necessity of downlink training in hybrid massive MIMO systems even in the presence of channel hardening. Future work in this direction can include the study of optimal power allocation schemes for this system, and the analysis of massive MIMO systems under a combination of different CSI impairments.

REFERENCES


